Probability-Based Modelling of Composites Manufacturing and its Application to Optimal Process Design

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ABSTRACT: The control of process-induced deformations in composite structures is important for cost-effective manufacturing. In recent years, significant advances have been made in predicting the average deformation behaviour, but little work has been done in predicting the variability, which results from uncertainties in both the raw material properties and the manufacturing process conditions. A probability-based approach is presented in this paper for predicting the variability of process-induced deformations. A two-dimensional finite element code, which deterministically simulates the various physical phenomena during processing of composite structures, is integrated with a first-order reliability analysis method to calculate the probability of the deformations exceeding a specified allowable tolerance. The methodology is demonstrated through two case studies. In the first study, a probabilistic description of the process-induced spring-in of a channel section is achieved and the effect of variability in material properties on the final channel angle is studied. In the second study, the optimal tool-shape for the channel section is determined by coupling reliability analysis with a simple cost model of the manufacturing process.

KEY WORDS: composite structure, manufacturing process, reliability analysis, process design, reliability-based optimisation.

INTRODUCTION

Deterministic Process Modelling

ADVANCED COMPOSITE MATERIALS are expensive on a per-weight basis compared to their metallic competitors. In many applications, particularly those that are weight sensitive, composite structures are superior from a performance perspective, but

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increasingly they must also be cost-competitive. This can be achieved by manufacturing large integrated complex-shaped components in one step, and thus trading off high raw material costs for lower manufacturing costs. This is feasible, and done routinely. However, it requires good control of process-induced deformations, since these may result in final component dimensions being outside the specification limits, resulting in mismatch and fit-up problems during assembly of the component into a larger structure. The residual stresses developed during the manufacturing process are responsible for the deformations, which may range from small, in-plane, and local spring-in angle of angular sections to large-scale warpage in larger structures.

Therefore, one of the major objectives during manufacturing is to maintain the residual deformations within pre-determined tolerance limits. Typically, to meet this objective, process variables such as tool geometry and process cycle are determined on the basis of experience and trial-and-error. This iterative procedure can be expensive, time-consuming and inefficient, especially when applied to large components. During the last two decades, many mathematical models of various aspects of the composite manufacturing process have been developed (e.g., [1,3,4,8,17,18,24,26,29,34]). More recently, researchers at The University of British Columbia have developed a software package (COMPRO) to simulate the complete process in an industrial environment, by integrating deterministic analytical and empirical models of the various physical phenomena within the framework of the finite-element method [20,22]. The accuracy and applicability of this deterministic software has been verified through comparison with experimental results and actual field data [11–13,19,21].

Sources of Variability

Engineering experience and laboratory tests [6] have shown that small variations in the raw composite material (prepreg) may lead to large variations in the cured composite part. Variations may also result from the variability involved in processing, including tooling, ply lay-up and autoclave loading. The combination of all these sources of variability results in variability of process-induced deformations in the finished composite part, as illustrated schematically in Figure 1. From an engineering point of view, a high or low variability reflects, respectively, poor or high quality of the processed component. In turn,

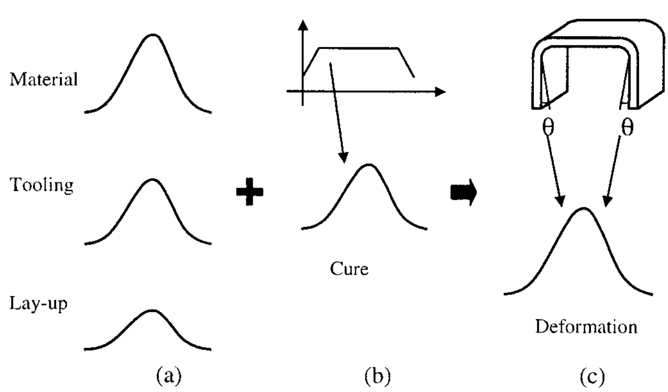

Figure 1. Schematic showing how variabilities in (a) raw material, tooling and lay-up combined with variabilities in (b) process conditions lead to variabilities in (c) the shape of the final processed structure.

variability of the individual components will significantly affect the variability of the completed mechanical assembly, both in geometry and functionality. This variability also makes it difficult for the manufacturer to adjust the process and possibly the input raw material to control the deformation. The consequences are significant: a composite part may be rejected from service because of large unwanted deformations, or at least additional costs may be incurred due to required shimming during assembly. Thus, there is a need to model the whole process on a probabilistic basis to provide a description for the process-induced deformation, considering all the uncertainties involved. Recently, Padmanabhan and Pitchumani [31] investigated the effects of process and material uncertainties on the variability of fill time and degree of cure in Resin Transfer Molding (RTM). There has also been work done on determining variability in process-induced deformations by simply performing parametric studies of the influence of input variables using deterministic analysis [12].

The probabilistic description is in general achieved by defining the intervening variables as *random* with given parameters and probability distributions. For example, the fibre volume fraction of a prepreg can be defined in terms of its mean value and standard deviation, with an associated distribution, e.g. Normal distribution. The determination of parameters and distribution for a random variable may be achieved by laboratory tests or surveys, or by engineering experience and judgement when such statistical information is lacking.

Effect of Variability on Assembly

As discussed earlier, composite material costs are significantly higher than metallic material costs, but the structure's costs, once manufactured and assembled, are comparable. Although published cost analyses of structures are rare, since they are both proprietary and specific to a company, some published information exists, e.g. [31]. In general, assembly costs are significant and most easily targeted for cost reduction. This is due to additional costs that are routinely incurred due to imperfection of the parts to be assembled, because their high stiffness and the small allowable clamp-up forces practically eliminates the possibility of forcing them into a desired shape. Therefore, either the lack of fit must be corrected by appropriate shimming, or the part has to be discarded. Both of these options have undesirable cost implications. As illustrated in Figure 2 and shown subsequently in this paper, shimming will be needed if the process-induced flange spring-in angle, θ, of a C-channel is less than an intermediate tolerance θ_s, while if $\theta_s < \theta < \theta_f$, then the part can be assembled without shimming. If the spring-in angle exceeds the allowable limit θ_f, the part must be discarded. The expected cost of shimming and failure can be predicted if one knows the probability of their occurrence and associated costs.

In addition, the variability of the individual components will also affect the variability of the completed mechanical assembly, both in geometry and in functional performance. This can be studied using tolerance analysis [7]. In these studies, the propagation of the variability in the assembly can be determined by using 2D and 3D tolerance analysis, based on the variability of the individual components. When applying the method of tolerance analysis to an assembly of composite components, one has to know a priori the variability of the individual components. Ideally, a complete procedure to evaluate the variability of an assembly should be implemented as shown in Figure 3. In this paper we will develop a methodology for step 2 in this flow chart.

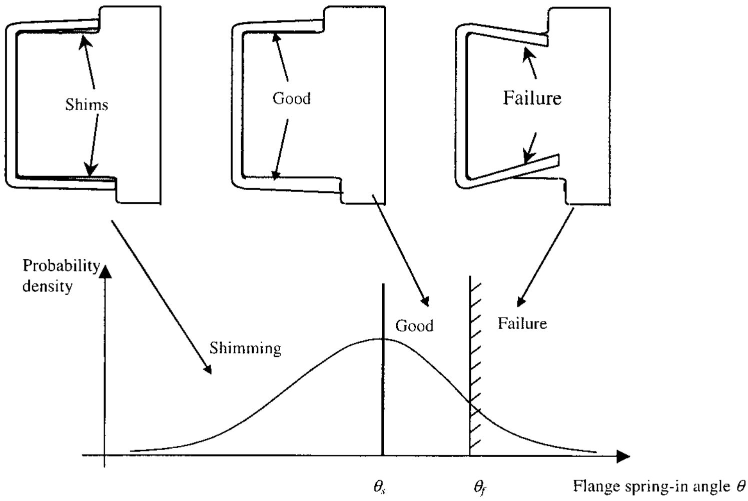

Figure 2. Consequences of process–induced flange spring-in angle of a C-channel assembly to a rigid mating structure.

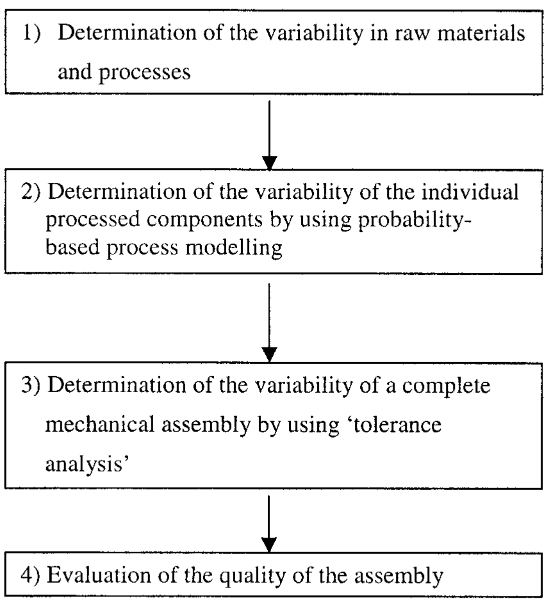

Figure 3. Flow chart of a complete variability analysis.

Optimal Design

Since the quality of the raw material and the process implementation may significantly influence the quality of composite parts, the manufacturer has the choice of either changing the raw material to be used in the process to lower the input material variability, or improving the quality of the process. For example, when using a material with low

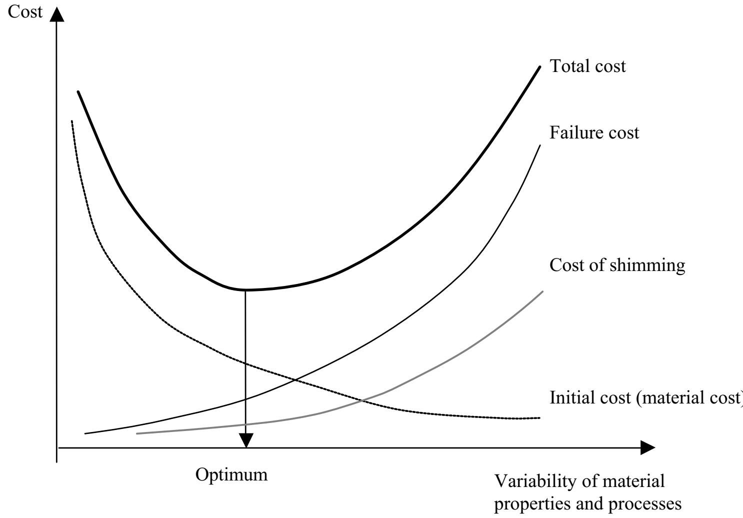

Figure 4. *Cost as a function of variability in composites manufacturing.*

quality the initial savings on the material is often offset by higher costs during structural assembly through 'shim-to-fit' and higher failure costs. On the other hand, choosing a material with high quality and high cost, the penalties during assembly may be negligible. Obviously, there is an optimal balance between these two extremes, at which the total expected cost, including the manufacturing cost, the expected cost of shimming and the expected cost of failure is minimum, as illustrated schematically in Figure 4.

The problem of minimising the total expected cost can be classified as a probability-based optimisation problem [15,27,28].

Probability-Based Modelling

To implement the type of approach described above, there is a need to model the composite manufacturing process on a probabilistic basis by considering all the uncertainties involved in the raw materials and the process. A probability-based model will provide the required statistical quantities of interest for a processed component, which can then be used to: (1) evaluate its quality, (2) calculate the variability of the assembly using tolerance analysis, or (3) perform probability-based design. Furthermore, it permits the engineer to find the most important parameters through probability-based sensitivity analysis.

To develop a probabilistic-based model, one first needs a deterministic model of the process to obtain the deformation of interest in terms of a set of parameters, ranging from material properties to process conditions, and then integrate that deterministic model with a probabilistic approach for implementation of a *reliability method*. The deterministic process model used in this study is COMPRO [18,20,22]. This model is combined with the reliability analysis software, RELAN [14], to quantitatively predict the uncertainty (probabilistic description) of process-induced deformations, which is then used in the application of probability-based optimisation to minimise the total expected cost as described earlier.

RELIABILITY ANALYSIS

The reliability of a general system is defined as the probability that it will perform as required under the given conditions within a given period of time. The performance of the system is normally controlled and influenced by several intervening variables, some representing material, mechanical and geometric properties and others characterising the external effects. From a probabilistic point of view, these variables are regarded as random and can be described in statistical terms, by specifying their mean values, standard deviations and distributions. The implementation of the reliability analysis is based on a description of the limit state of interest by a performance or limit-state function $G(\boldsymbol{x})$ where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ is an n-dimensional vector of random variables. Some of these may affect the demand or response of the system, denoted by D, while the others may influence the system capacity or tolerance C to withstand the demand. The performance function G may be written in terms of C and D as

$$G = C - D \tag{1}$$

In composite manufacturing, the system can be defined as the entire process, the demand as the process-induced deformation, and the capacity as the allowable deformation limit. Thus, the process will not perform as intended (part must be discarded or needs shimming) if the combination of the random variables (e.g., material properties and process conditions) results in $G < 0$. The corresponding probability of such an event, $\mathrm{Prob}(G < 0)$, is called the probability of failure, P_f, as shown in Figure 5. Conversely, the combination of the random variables resulting in $G > 0$ will make the system perform as required (produce good parts) and the corresponding probability, $\mathrm{Prob}(G > 0)$, is termed the reliability P_r. The condition when $G = 0$ is a limit-state that defines the boundary between failure and survival and the corresponding surface in the n-dimensional space of random variables, $G(\boldsymbol{x}) = 0$, is called the limit-state failure surface. The probability of failure is complementary to the reliability:

$$P_r + P_f = 1 \tag{2}$$

For the problems to be studied in this paper, the limit-state function Eq. (1) can be rewritten as

$$G = C - D(x_1, x_2, \ldots, x_n) \tag{3}$$

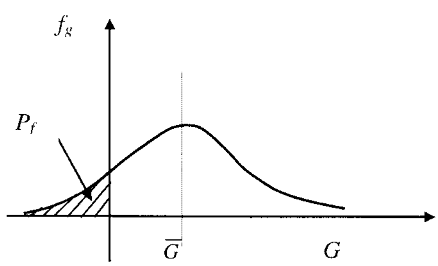

Figure 5. *Schematic of probability density function for the performance function G. Shaded area corresponds to the probability of failure.*

where C is a specified allowable deformation for the processed composite part and D is the process-induced deformation of the part, which is a function of the parameters, including random variables, that drive the process. The calculation of the maximum deformation is accomplished by running a deterministic finite element model such as COMPRO. To enable a reliability analysis, this deterministic model must be able to provide the output of interest and be integrated with the reliability analysis software. The integration of the deterministic model and the reliability analysis forms the basis of the probability-based modelling approach proposed in this paper.

The probability of failure can be obtained by calculating the probability of the event $G < 0$. Since, in general, there could be a number of random variables involved in G, the exact calculation requires the joint probability density function of all random variables be integrated over the failure region $G < 0$,

$$P_f = \int_{G<0} f_{123\ldots n}(x_1, x_2, \ldots, x_n)\, dx_1 dx_2 \cdots dx_n \tag{4}$$

However, this exact approach is difficult to apply since the joint probability density function, f, is unknown and very difficult to determine. An alternative is the straight-forward Monte Carlo method which is simple to implement and can converge to the exact solution. However, this technique is computationally demanding, especially when dealing with a low probability of failure. For example, if the probability of failure is 10^{-5}, the performance function must be evaluated 100,000 times in order to observe, on average, one outcome where $G < 0$. Particularly, if the G function must be obtained by running a separate computer program, for example a nonlinear analysis with time integration, the direct simulation approach will be computationally intensive.

A second alternative is the use of approximate methods that have been developed during the last three decades, such as FORM (First Order Reliability Method [17,32]), and SORM (Second Order Reliability Method [5,9,33]), which are based on the calculation of the reliability index β. From this index, the probability of failure, P_f, and the reliability, P_r, can be estimated approximately using the Standard Normal probability distribution function $\Phi\ (\cdot)$:

$$P_f = \Phi(-\beta) \tag{5}$$

and

$$P_r = 1 - \Phi(-\beta) = \Phi(\beta) \tag{6}$$

The details of the FORM procedure are provided in Appendix A.

BASIS OF DETERMINISTIC ANALYSIS

COMPRO is a two-dimensional plane-strain finite element code that simulates the various phenomena involved in the processing of composite structures [18–22]. It incorporates the effects of heat transfer, resin flow-compaction, resin cure kinetics, resin cure shrinkage and stress-deformation development. A wide range of problems and configurations can be modelled, including multi-directional laminates, fabrics, tool-part interfaces, honeycomb, tool, inserts and different autoclaves. The model predicts the temperature of the curing part, resin degree of cure, resin viscosity, fibre volume fraction, resin pressure, part thickness, and process-induced deformation. The overall structure of the process model is an integration of a series of 'sub-models' or 'modules' that are analysed independently and in sequence over time. Of particular interest here are the thermochemical module and the stress-deformation module, which work together to predict the process-induced deformations.

The thermochemical module is used to calculate the distribution of component temperature T and the degree of resin chemical conversion α (degree of cure). Two sub-models, heat transfer and resin reaction kinetics, are considered in the analysis, permitting the prediction of current temperature and degree of cure, two fundamental 'state' variables used to characterise the properties of the composite material during the entire process. Based on a finite element formulation and time integration, the governing thermochemical equations can be solved to provide the current temperature and degree of cure used by the other modules throughout the process.

The stress-deformation module is responsible for modelling the internal stresses and stress-induced deformation developed within a component during processing. Through integration of this module with thermochemical module, the model can examine all five major sources of process-induced stress and deformation identified in the literature [23]. These are: (1) thermal expansion, (2) resin cure shrinkage, (3) gradients in component temperature and resin degree of cure, (4) resin pressure gradients (resulting in resin flow), and (5) mechanical constraints due to tooling.

The system of algebraic equations for the discretised problem is obtained by invoking the principle of stationary potential energy in the standard manner. By solving these equations, the nodal displacements for the complete component or structure can be determined, allowing the calculation of the process-induced deformation.

CASE STUDY 1: VARIABILITY OF FLANGE SPRING-IN ANGLE OF A COMPOSITE CHANNEL SECTION

Description of the Composite Component

A C-shaped composite laminate, shown in Figure 6, is considered. Among the various outputs of COMPRO, the process-induced spring-in angle θ, as illustrated in Figure 7, is of particular interest here. The laminate consists of 16 layers of prepreg with a $[0/45/-45/90/0/45/-45/90]_S$ lay-up. The laminate is processed on an aluminum tool with a single-hold cure cycle as shown in Figure 8. In the deterministic model, the autoclave temperature is increased to a target level of 350°F, at a heating rate of 5°F per minute, held for 2 h, and then cooled down to room temperature at a rate of -6°F per minute. The autoclave is pressurised to 65 psi (0.448 MPa) during the entire cycle.

The composite material is Hercules AS4/8552, a unidirectional carbon-fibre epoxy prepreg. Its relevant properties from a process modelling viewpoint include fibre volume fraction, density, specific heat capacity, conductivity, thermal expansion, resin cure kinetics, resin cure shrinkage and elastic material constants. The tool material is 6061-T6 aluminum with properties taken from a handbook. A detailed description of the material properties used in the model is provided in Johnston [21].

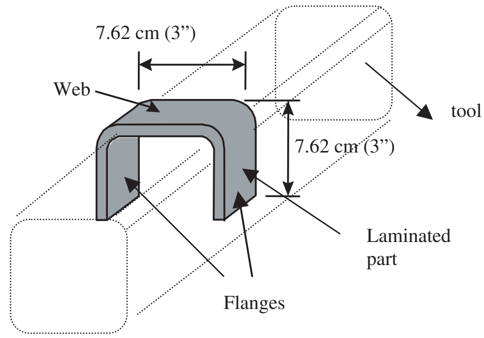

Figure 6. *Schematic of the C-shaped part and tool used in the case study.*

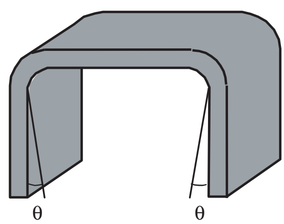

Figure 7. *Schematic showing the process–induced spring-in of a C-shaped laminate.*

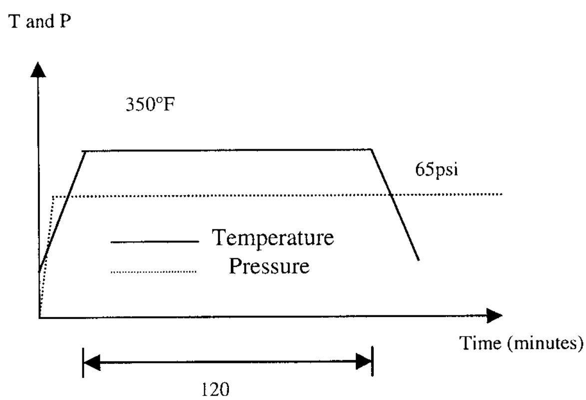

Figure 8. *Cure cycle used in the case study.*

Uncertainty Characterisation

In probability-based process modelling, the sources of uncertainty should first be determined. These uncertainties may be associated with the input material properties, tool geometry, ply lay-up and autoclave loading as well as the accuracy of the deterministic process model.

There are many parameters involved in the COMPRO model, ranging from material properties and tooling to the cure cycle. To simplify the problem, only those parameters

Table 1. Random variables used in Case Study 1.

Variable	Mean Value	COV	Distribution
RCS (Associated with resin cure shrinkage)	0.1	0.1	Normal
VF (Volume fraction of fiber)	0.57	0.1	Normal
CTE3 (through thickness coefficient of thermal expansion)/°C	2.86×10^{-5}	0.1	Normal
T (Hold temperature) °*F*	350.0	0.01	Normal
R_n (Model error)	1.0	0.1	Normal

exhibiting significant effect on the process-induced deformations will be treated as random variables.

In the current case study, the following assumptions are made: (1) the variability in composite materials and in autoclave loading are taken into account; (2) tooling, ply lay-up and bagging are assumed to be free from uncertainties and (3) model error is considered by introducing a random variable R_n with a mean value of 1.0 (which represents no error) and standard deviation of 0.1. Based on these assumptions and previous sensitivity analyses [21,23], five parameters, as listed in Table 1, were chosen as random variables. At this stage of our understanding, we are trying to emphasise the methodology rather than the details of the case study.

As shown in Table 1, all five variables are assumed to be Normally distributed. In the absence of statistical data for these random variables, the nominal values of the material properties are used as the mean values for resin cure shrinkage (RCS), fibre volume fraction (VF), and through-thickness coefficient of thermal expansion (CTE3) with an associated coefficient of variation (COV) of 0.1. The mean value of the hold temperature T is taken to be 350°F and its coefficient of variation is taken as 0.01. This ensures that the temperature will fall within the range of $350° \pm 10°$F with a probability of 99.7%.

Limit-State Function

The process-induced spring-in angle, θ, is assumed to be the only deformation of interest here. Therefore, the performance function G can be defined as:

$$G = \theta_0 - \theta \tag{7}$$

where θ is the spring-in angle predicted by COMPRO, which is a function of the random variables and other deterministic parameters. θ_0 is the given tolerance limit, which depends on the actual requirements for the assembly. The probability of the event $G < 0$ is then the probability that the actual spring-in angle will exceed θ_0.

Results

The objective of this case study is to predict the variability (probabilistic description) of the process-induced deformation (spring-in angle) for a specified manufacturing process. This can be done by calculating the probability that $G > 0$, or Prob($\theta < \theta_0$), for a given tolerance limit θ_0 using reliability analysis, RELAN, in conjunction with COMPRO.

A discrete distribution is first obtained through reliability analyses for different values of θ_0. These values are then fitted by a mathematical function $F(\theta_0)$ to provide a complete probabilistic description for the deformation. The probability of the deformation exceeding any given tolerance θ_0 can then be obtained by calculating $1 - F(\theta_0)$.

For the statistical quantities listed in Table 1, the RELAN/COMPRO results, which describe the discrete distribution of the spring-in angle θ, are presented in Table 2. To provide a complete description of the variability of the spring-in angle, the analytical results shown in Table 2 were fitted by several probabilistic models using the least-squares method. It was found that the best fit was a normal distribution with a mean value of 1.50° and a standard deviation of 0.17°. Figure 9 shows both the analytical results and the fitted normal distribution function. This predicted probabilistic model is essential to evaluate not only the quality of the composite product, but also to carry out the tolerance analysis of an assembly system composed of processed components and probability-based optimisation, addressed in Case Study 2.

To investigate the effect of the statistics of the random variables on the probability function, sensitivity analyses were performed for different mean values and coefficients

Table 2. Probability of G > 0 ($\theta \le \theta_0$) for various tolerances.

θ_0 (°)	*Prob*(G > 0)	θ_0 (°)	*Prob*(G > 0)
1.0	0.00123	1.50	0.52244
1.2	0.03818	1.55	0.63599
1.25	0.07207	1.60	0.73785
1.30	0.12490	1.65	0.82168
1.35	0.19949	1.70	0.88656
1.40	0.29498	1.80	0.95963
1.45	0.40496	2.00	0.99729

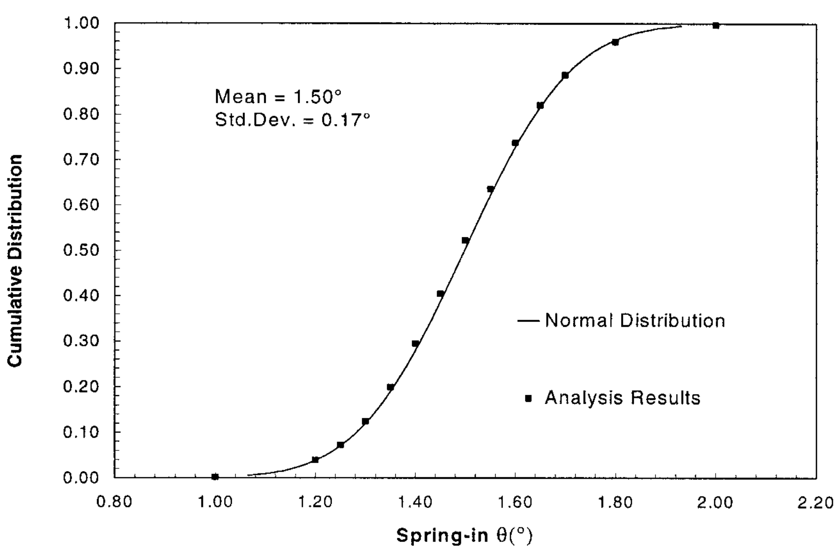

Figure 9. *Predicted distribution of the spring-in angle, θ.*

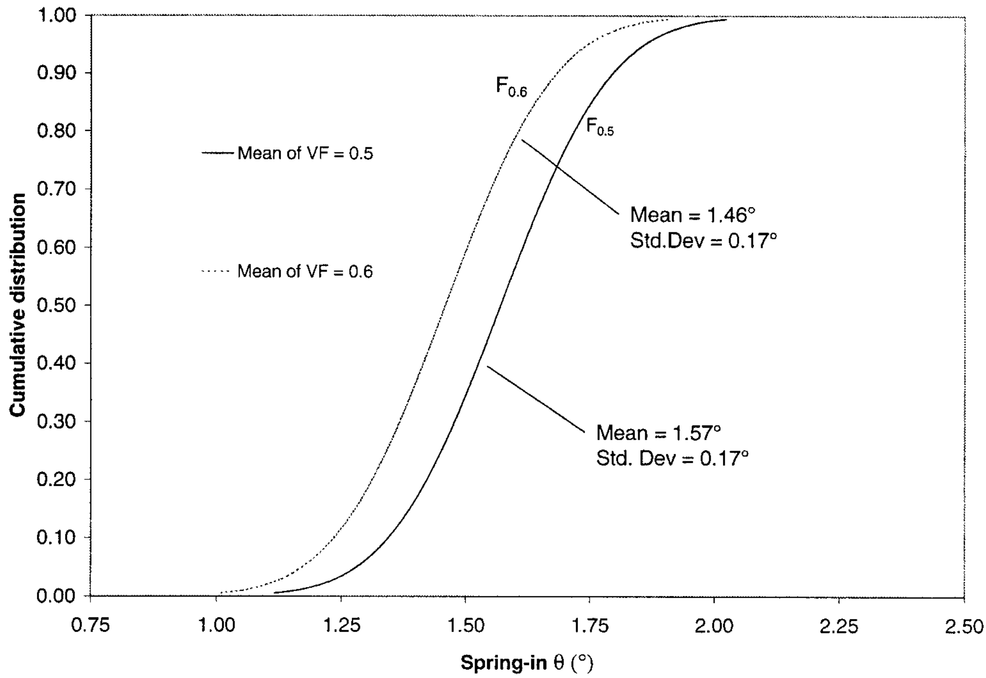

Figure 10. *Predicted probability distributions of the spring-in angle for two different mean values of fibre volume fraction VF.*

of variability as described below. The sensitivity of the probability function to the mean value of the fibre volume fraction, *VF*, was studied first. The probability functions $F_{0.5}$ and $F_{0.6}$ were obtained for mean values of 0.5 and 0.6, respectively, as shown in Figure 10. The statistics of the other random variables remained the same as the baseline values listed in Table 1. It is seen that increasing the mean value of *VF* from 0.5 to 0.6 will shift the probability distribution to the left (the mean value changes from 1.57 to 1.46°) without varying its shape (standard deviation), resulting in $F_{0.6} > F_{0.5}$. In other words, for a given tolerance θ_0, an increase in the mean value of the fibre volume fraction will increase the probability of survival, Prob($G > 0$).

To investigate the effects of the variability in the random variables, the COV of the three variables associated with material properties (RC, VF, CTE3) were changed simultaneously from 5 to 20%. The COV of the hold temperature and the COV of the model error remained the same as the baseline. The probability distribution functions $F_{0.05}$ and $F_{0.2}$ for the flange spring-in angle are shown in Figure 11. It is seen that a decrease of the COVs for the three material-related variables from 20 to 5% will decrease the overall variability of the spring-in angle θ from 0.23 to 0.16° without shifting the mean value. It is also seen that the influence of the three COVs on the probability of failure depends strongly on the magnitude of the tolerance limit θ_0, or the probability of failure being calculated. For example, with $\theta_0 = 1.7°$, the probability of failure $P_f(1 - F(\theta_0))$ changes from 0.186 at COVs = 20% to 0.093 at COVs = 5%. However, with $\theta_0 = 1.5°$ no significant change in P_f is observed between COVs = 20% and COVs = 5%; in both cases $P_f = 0.5$. The opposite trend is seen when, for example, $\theta_0 = 1.3°$.

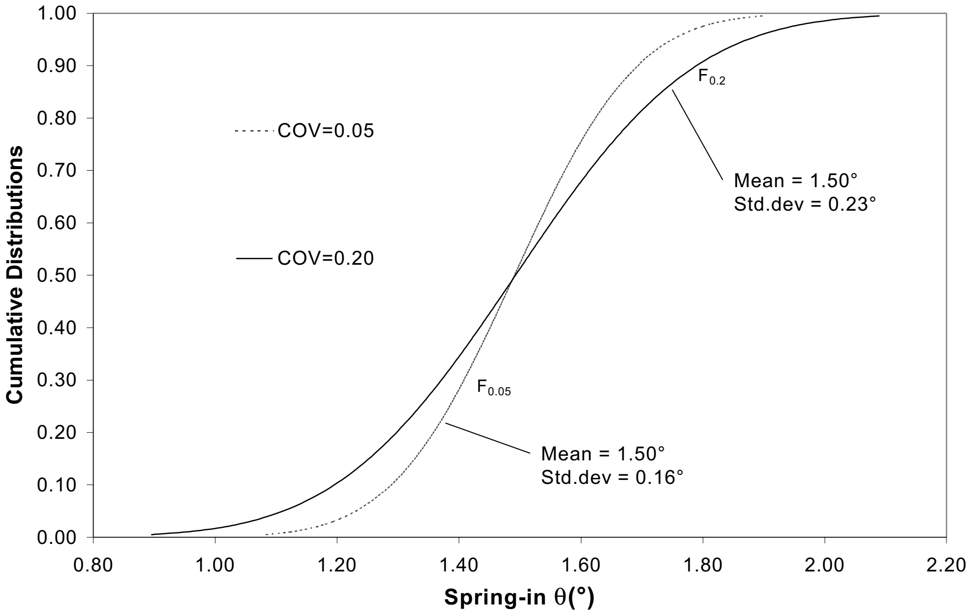

Figure 11. *Predicted probability distributions for the spring-in angle for two different coefficients of variation of the three material-related properties.*

CASE STUDY 2: PROBABILITY-BASED OPTIMAL TOOL DESIGN

A cost-oriented optimisation problem in process design is now discussed. It takes into account the total expected cost involved in both the manufacturing process and the structural assembly.

The Problems of Interest

Figure 12(a) shows a composite channel section (Part A) that is to fit a mating structure (Part B). A perfect fit requires that α_{part}, the final angle of the cured Part A equals $\alpha_{ms,}$ the angle of Part B for assembly purposes. To achieve this, Part A is to be made on a tool with a properly designed angle α_{tool}. Due to the process-induced spring-in angle, θ, α_{part} can be expressed as

$$\alpha_{\text{part}} = \alpha_{\text{tool}} - \theta \tag{8}$$

Because of the uncertainties involved in the raw materials and in the process, the spring-in angle θ is, in fact, a random variable and its variability depends on the variability of the raw materials and the process. Figure 13 shows four possible scenarios that can occur in the assembly of Part A and Part B. Let θ_1 be a small angle denoting the tolerance limit for an acceptable part and θ_2 be the angle that corresponds to the limit where the assembly can be accomplished through shimming. Furthermore, to facilitate the following

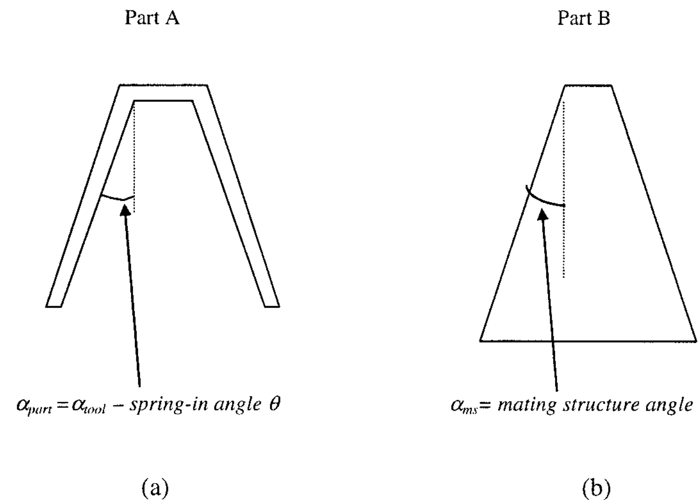

Figure 12. *Schematics of (a) Part A and (b) Part B, mating structure.*

study, a design parameter θ_d is defined as the difference angle between the mating structure and the tool:

$$\theta_d = \alpha_{\text{tool}} - \alpha_{ms} \tag{9}$$

The above four assembly events can be summarised as follows:

(a) When $\alpha_{\text{part}} < \alpha_{ms}$ (or $\theta > \theta_d$), the α_{part} is too small to fit the mating structure and the part fails, with a corresponding cost of failure.
(b) When $\alpha_{ms} < \alpha_{\text{part}} < \alpha_{ms} + \theta_1$ (or $\theta_d - \theta_1 < \theta < \theta_d$), the composite part is acceptable to be used without extra assembly costs.
(c) When $\alpha_{ms} + \theta_1 < \alpha_{\text{part}} < \alpha_{ms} + \theta_2$ (or $\theta_d - \theta_2 < \theta < \theta_d - \theta_1$), the part can still be used but extra cost is incurred to shim the gap.
(d) When $\alpha_{\text{part}} > \alpha_{ms} + \theta_2$ (or $\theta < \theta_d - \theta_2$) α_{part} is too large to be shimmed. The part fails with a corresponding cost of failure.

The probabilities corresponding to the above events are defined as follows:

- $P_{f1} = \text{Prob}(\theta > \theta_d)$, the probability of making a part that leads to an interference type failure.
- $P_r = \text{Prob}(\theta_d - \theta_1 < \theta < \theta_d)$, the probability of making an acceptable part.
- $P_s = \text{Prob}(\theta_d - \theta_2 < \theta < \theta_d - \theta_1)$, the probability of making a part that requires shimming.
- $P_{f2} = \text{Prob}(\theta < \theta_d - \theta_2)$, the probability of making a part that leads to excessive clearance (failure).

A graphic illustration of the above probabilities is shown in Figure 14. The probabilities must satisfy the following relationship:

$$P_{f1} + P_r + P_s + P_{f2} = 1.0 \tag{10}$$

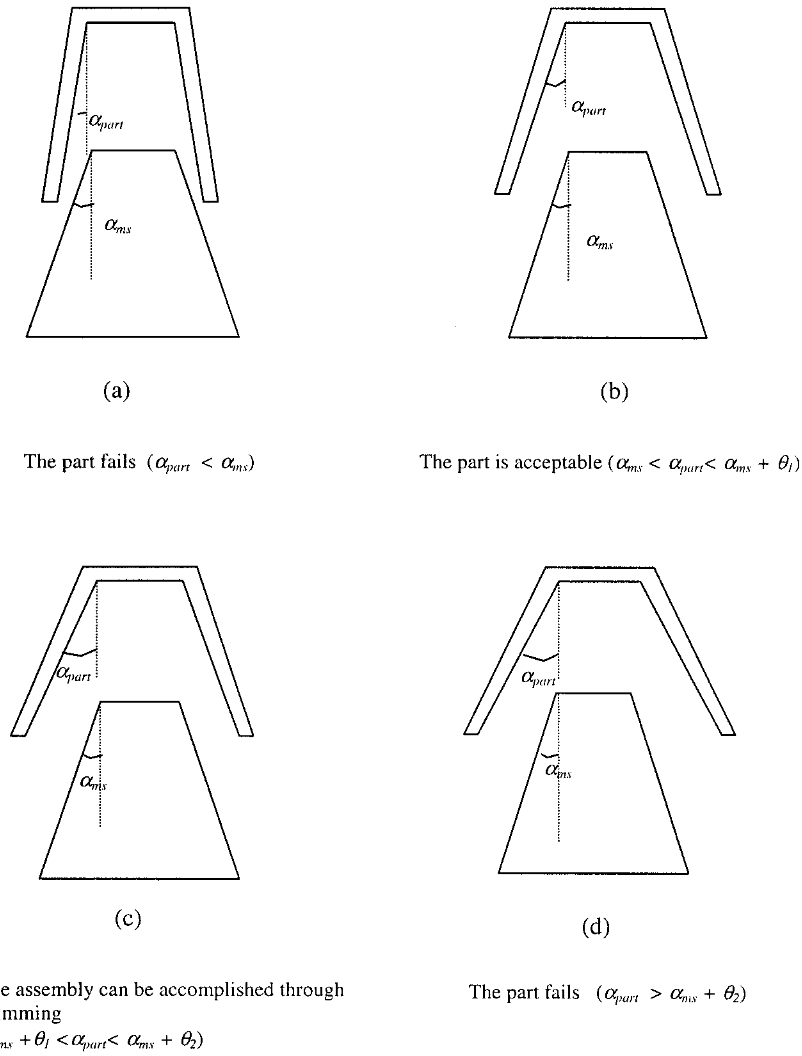

Figure 13. *Illustration of the four possible events in the assembly of the part to the mating structure.*

For the given material properties and process conditions, the probability description of the process-induced spring-in angle, θ, can be predicted by performing a reliability analysis, as discussed in Case Study 1. The mean value and coefficient of variation of θ are dependent on the nominal design values and the variability of raw materials and process conditions. The objective of this case study is to minimise the total expected cost through design of the tool angle α_{tool}.

Cost Model

We will now develop a simple cost model for the assembly problem. If C_s is the cost of shimming and C_{f1} and C_{f2} are the costs of failure, for interference and excessive

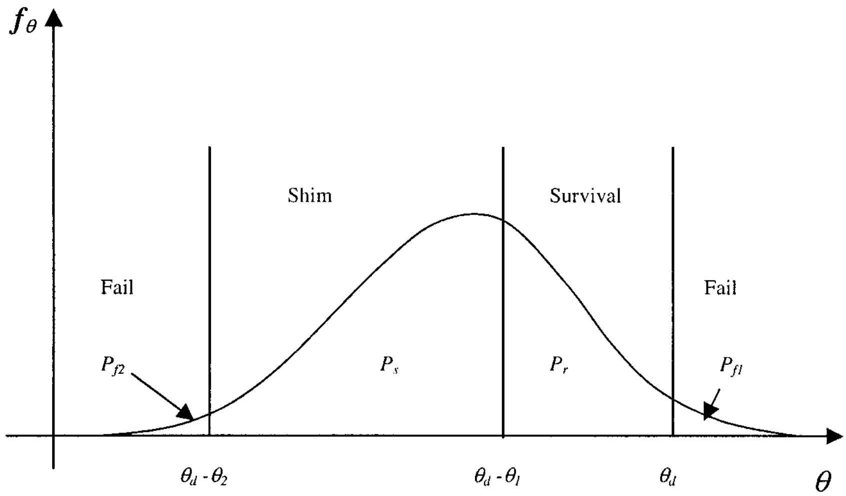

Figure 14. *Illustration of the probabilities of the four assembly events.*

clearance, respectively, the expected shimming cost and the expected costs of failure are defined, respectively as given by:

$$\text{Expected cost of shimming (Ecs)} = C_s P_s(\theta_d) \tag{11}$$

$$\text{Expected cost of failure due to interference } (\text{E}_{\text{CF1}}) = C_{f1} P_{f1}(\theta_d) \tag{12}$$

$$\text{Expected cost of failure due to excessive clearance } (\text{E}_{\text{CF2}}) = C_{f2} P_{f2}(\theta_d) \tag{13}$$

where θ_d is the design parameter associated with the tool angle as defined in Equation (9).

Using the reliability-based optimisation technique, minimum cost is achieved by minimising the objective function:

$$E_t = C_q + C_s P_s(\theta_d) + C_{f1} P_{f1}(\theta_d) + C_{f2} P_{f2}(\theta_d) \tag{14}$$

where E_t is the total expected cost of assembly and C_q is the fabrication cost associated with the cost of materials. It can be formulated as

$$C_q(\sigma_\theta) = C_0 + f(\sigma_\theta) \tag{15}$$

where C_0 is a constant, considered as the basic cost, and $f(\sigma_\theta)$ is a function related to the quality of the process and materials and is typically inversely proportional to the variability σ_θ (implying that the lower the variability, the higher the cost). The cost of shimming results from addressing the mismatch between the parts in assembly, and includes the cost of shimming materials as well as labour. Although the cost of shimming depends on the amount of mismatch, to simplify the study it is assumed that this cost, C_s, is a constant and is taken as the average of the costs for all gap sizes. The estimate of the costs of failure C_{f1} and C_{f2} includes the costs associated with materials, labour and the impact on the overall assembly process.

Optimisation

With the total expected cost E_t expressed as a function of the design parameters, the minimisation of the total cost can be performed through a nonlinear programming technique, COMPLEX [2]. With a given feasible range of θ_d, the optimisation problem is simply stated as:

$$\text{Minimise}: \quad E_t(\theta_d, \sigma_\theta) \tag{16}$$

$$\text{Subject to}: \quad \theta_L < \theta_d \leq \theta_U \tag{17}$$

where θ_L and θ_U are lower and upper bounds of θ_d, respectively.

For given costs of failure, shimming and material, the optimal θ_d is calculated and used to determine the appropriate tool angle, α_{tool}.

Results

From Equations (16) and (17), the problem is stated as

$$\text{Minimise}: \quad E_t = C_q + C_s P_s(\theta_d) + C_{f1} P_{f1}(\theta_d) + C_{f2} P_{f2}(\theta_d) \tag{18}$$

$$\text{Subject to}: \quad 0.0^\circ < \theta_d \leq 3.0^\circ \tag{19}$$

Thus, using Figure 14, Equation (18) can be rewritten as:

$$E_t = C_q + C_s[F_\theta(\theta_d - \theta_1) - F_\theta(\theta_d - \theta_2)] + C_{f1}(1 - F_\theta(\theta_d)) + C_{f2} F_\theta(\theta_d - \theta_2) \tag{20}$$

where C_q is a constant conditional on the σ_θ being considered. Since actual costs are hard to obtain, for the purposes of this study we will use qualitative dimensionless costs. Let us assume that the shimming cost $C_s = 4$, the costs of failure $C_{f1} = C_{f2} = 40$, the cost of materials $C_q = 1.25$ and $\theta_1 = 0.2^\circ$ and $\theta_2 = 0.6^\circ$. From Case Study 1, we know that the probability distribution of the spring-in angle, F_θ, is Normally distributed with mean value $M_\theta = 1.5^\circ$ and standard deviation $\sigma_\theta = 0.17^\circ$. Carrying out the minimisation using the baseline case as given in Table 1, the optimal design angle was found to be $\theta_d = 1.783^\circ$, and the total expected cost $E_t = 7.04$.

To investigate the effect of variability of the spring-in angle θ on total expected cost, sensitivity analyses were carried out for three different models for F_θ. They are referred to as Models A, B and C with a coefficient of variation of the spring-in angle, COV = 4, 10 and 20%, respectively. Here, we assume that the nominal raw material properties and the process conditions remain the same as in the base-line case but that the variability involved in both material properties and process conditions vary. As discussed before, this change will only reflect the COV of the spring-in angle or its standard deviation. Table 3 shows their statistical parameters, distributions and the corresponding cost C_q. Table 4 presents the optimal results. Figures 15–17 illustrate graphically the optimal probabilities, optimal costs and the relationship between the costs and θ_d.

From Figures 15 and 16, it can be seen that the optimal expected costs E_{f2}, E_s and E_{f1} are significantly controlled by the variability of material properties and process conditions.

Table 3. Statistical parameters for different probabilistic models considered.

Model	COV (%)	C_q	M_θ (°)	σ_θ (°)	Distribution
A	4.0	5.0	1.5	0.06	Normal
B	10.0	2.0	1.5	0.15	Normal
C	20.0	1.0	1.5	0.3	Normal

Table 4. Optimal results.

			Shimming		Failure 1		Failure 2	
Model	E_t	θ_d	E_{CS}	P_S	E_{CF1}	P_{f1}	E_{CF2}	P_{f2}
A	6.03	1.640	0.66	0.1647	0.37	0.009	0.0	0.0
B	6.64	1.780	2.72	0.6797	1.30	0.0326	0.62	0.0155
C	15.58	1.784	1.86	0.4637	6.89	0.1723	5.83	0.1457

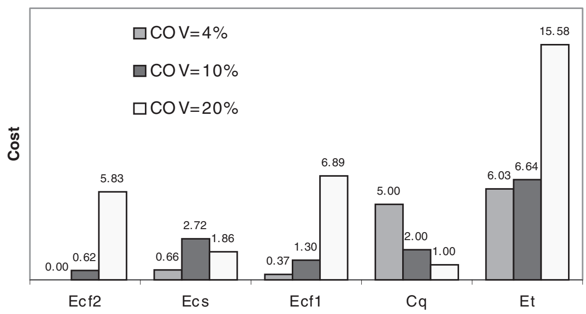

Figure 15. *Optimal costs for different material variability levels.*

After optimisation, the probabilities of failure, shimming and survival are redistributed by adjusting the θ_d so that the total expected cost reaches its minimum. From Figure 16, it is also seen that increasing the quality may reduce or even eliminate the probability of failure, resulting in drastic reduction in the expected cost. The effects of variability on the optimal total expected cost E_t depends not only on the variability-related cost C_q, but also on the quality level σ_θ (or, COV). For example, by decreasing the COV from 10 to 4%, the total expected cost is only reduced from 6.64 to 6.03, while decreasing the COV from 20 to 10%, results in a significant saving on total expected cost.

From Figure 17, it is seen that the contribution of the expected costs of failure to the total expected cost varies with the variability throughout all the feasible range. For the model A (COV = 4%), the total expected cost reaches its minimum at $\theta_d = 1.64°$. However, this design angle may not be used in reality, since the total expected cost is very sensitive to smaller values of θ_d. In this problem, it would be safer to choose $\theta_d = 1.7°$ or $\theta_d = 1.8°$. For the model B (COV = 10%), the optimal design is $\theta_d = 1.78°$. The figure shows that the total expected cost is insensitive to small changes of θ_d around $\theta_d = 1.78°$. For model C (COV = 20%), the optimal design is $\theta_d = 1.784°$. The probabilities of failure dominate the total expected cost and it is quite insensitive to small changes of θ_d around the optimal value.

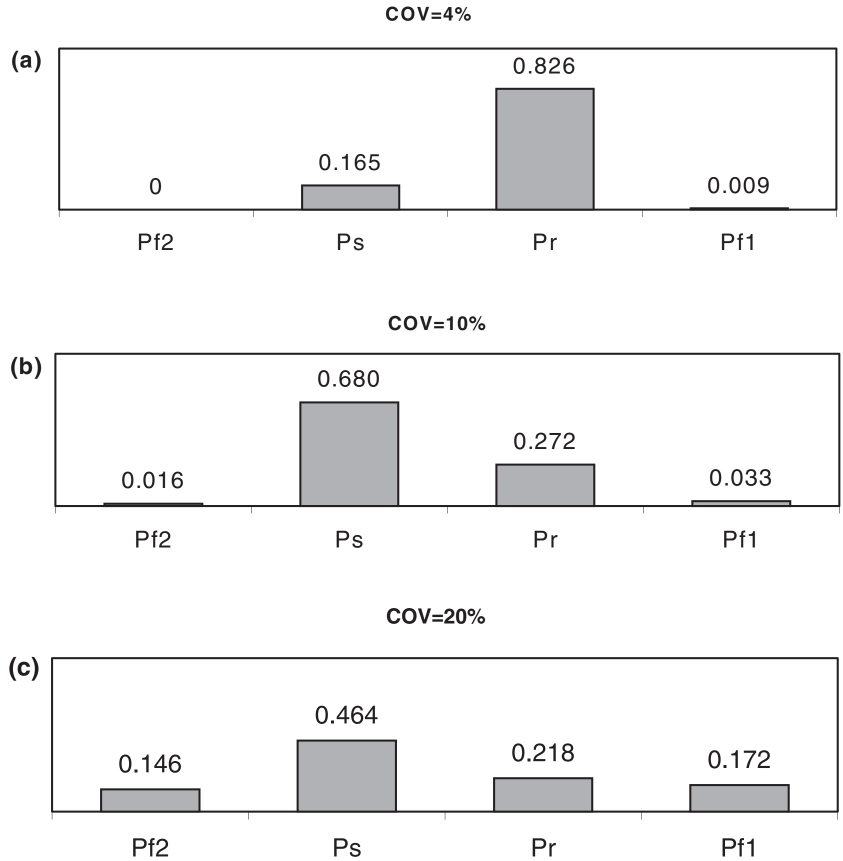

Figure 16. *Optimal probabilities for different material variability levels.*

SUMMARY AND CONCLUSIONS

Issues and problems of uncertainties affecting composite manufacturing have been raised and approached. In conjunction with COMPRO, a deterministic numerical model for simulating the composite manufacturing process, a reliability method has been applied to calculate the probability function of the process-induced deformation of a composite component. The uncertainties involved in the material properties, process implementation and numerical model have been taken into account. To illustrate the approach and its applications, two case studies were presented. The first demonstrated a procedure to obtain the probability function of the spring-in angle for a C-shaped composite component. The predicted probabilistic model, quantified by the mean and standard deviation and type of distribution, is very useful for assessing the quality of the components in manufacturing and further to predict the variability of an assembly, thus quantifying its quality with sufficient statistical data. The second example raised a problem of optimal design in the presence of variability in the part's residual deformation. Considering the costs of failure, shimming and manufacture and materials, the tool angle was designed to minimise the total expected cost.

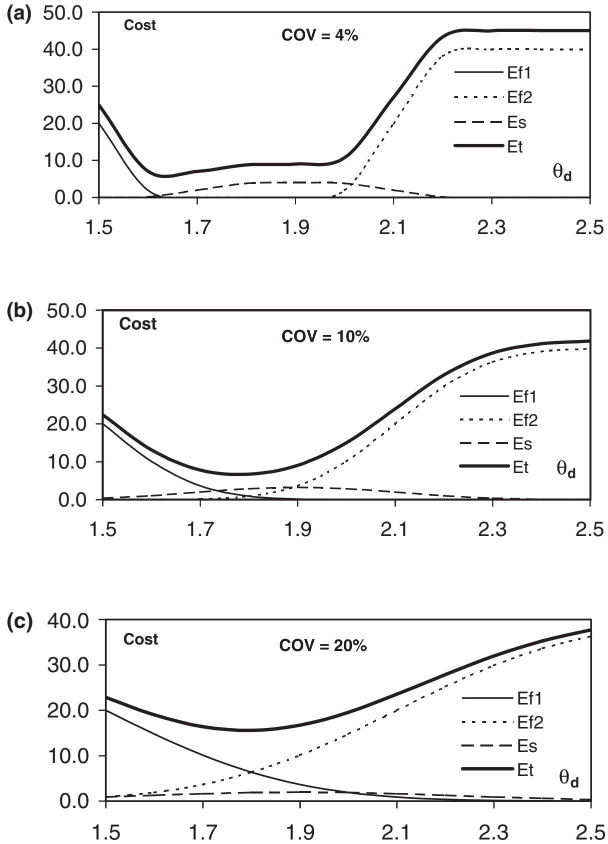

Figure 17. *Cost vs. θ_d for different material variability levels.*

It should be noted that it is a difficult task to collect enough statistical data on both raw materials and process conditions due to associated costs and complications. Nevertheless, the probability-based approach is the only logical and rational way to deal with the uncertainties that are inevitable in the manufacturing process.

This paper was only intended to provide a practical methodology to different problems in the manufacturing of composites when the uncertainties of the intervening variables have to be considered. With sufficient information on materials, processing implementation and analysis model, real problems can be systematically approached in a practical manner. In principle, the reliability-based approach presented here is capable of quantifying the probability of non-performance (failure) of a manufactured composite component regardless of the criteria that are used in the deterministic model to signal failure. The particular failure condition considered in this study was excessive process-induced distortions of manufactured composite components as they affect assembly and its associated costs. Failure criteria that account for other forms of potential defects during manufacturing such as delaminations, under- or over-curing of the resin, excessive residual stresses, etc., can conceivably be implemented in the deterministic model (COMPRO), which combined with the reliability model can result in a more general tool for studying the quality of a manufactured composite part from a probabilistic viewpoint.

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APPENDIX A

In order to avoid computationally intensive simulation, an efficient procedure called First Order Reliability Method (FORM [16,32]) was developed in the early 1970s and has been modified and extended since then. For the last two decades, these computer-based methods have been widely used in various engineering fields.

The objective of FORM is to find an approximation to the integral in Equation (4) through the calculation of a reliability index β. Given statistical information on the random variables $x_i (i = 1, 2, \ldots, n)$ and the limit-state function $G = G(x_1, x_2, \ldots, x_n)$, the reliability P_r or probability of failure P_f can be obtained by using the following FORM algorithm. First, assume that the random variables x_i are Normally distributed and uncorrelated (independent). A new set of random variables u_i are defined in the Standard Normal space and are obtained through the mapping

$$u_i = \frac{x_i - \bar{x}_i}{\sigma_{x_i}} \quad (i = 1, 2, \ldots, n) \tag{a-1}$$

where $\bar{x}_i$ is the mean value of x_i and σ_{x_i} its standard deviation. The origin $\boldsymbol{u} = 0$ is the mapping of the mean vector of $\boldsymbol{x}$.

The reliability index β is the minimum distance between the origin $\boldsymbol{u} = 0$ and the limit-state surface $G(\boldsymbol{u}) = 0$, as illustrated in Figure a-1 for the case of two variables u_1 and u_2. Taking advantage of this geometric interpretation of β, several iterative computer algorithms have been proposed and are well established to find this minimum distance [25]. The point $\boldsymbol{u}^*$ on the limit-state surface which is the closest point to the origin is called

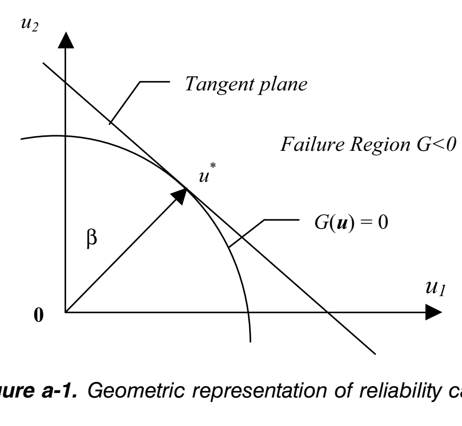

Figure a-1. *Geometric representation of reliability calculation.*

the design point, representing the most likely combination of random variables at failure. For Standard Normal, uncorrelated variables and a linear function $G(\boldsymbol{u})=0$, it can be proven that Equations (5) and (6) are exact. In the general case, this use of the index β gives an approximate estimate of P_f.

The reliability index is then obtained by minimising the distance between the origin and the failure surface, $G(\boldsymbol{u})=0$. Thus, letting

$$\Psi = \boldsymbol{u}^T\boldsymbol{u} + \lambda G(\boldsymbol{u}) \tag{a-2}$$

where λ is a Lagrange multiplier, the optimum solution can be reached if the vector $\boldsymbol{u}$ meets the following conditions:

$$\nabla_{\boldsymbol{u}}\Psi(\boldsymbol{u},\lambda) = 0 \tag{a-3}$$

and

$$\frac{d\Psi}{d\lambda} = G(\boldsymbol{u}) = 0 \tag{a-4}$$

where $\nabla_u(\cdot)$ denotes the gradient with respect to $\boldsymbol{u}$.

From Equations (a-2) and (a-3),

$$\boldsymbol{u} = -\frac{\lambda}{2}\nabla_u G \tag{a-5}$$

Multiplying both sides of Equation (a-5) by $\nabla_u G^T$ (i.e. transpose of the gradient vector $\nabla_u G$),

$$\frac{\lambda}{2} = -\frac{\nabla_u G^T\boldsymbol{u}}{\nabla_u G^T\nabla_u G} \tag{a-6}$$

and substituting Equation (a-6) into Equation (a-5)

$$\boldsymbol{u} = \frac{\nabla_u G^T\boldsymbol{u}}{\nabla_u G^T\nabla_u G}\nabla_u G \tag{a-7}$$

Therefore,

$$\beta^2 = \boldsymbol{u}^T\boldsymbol{u} = \left(\frac{\nabla_u G^T\boldsymbol{u}}{\nabla_u G^T\nabla_u G}\right)^2 \nabla_u G^T\nabla_u G = \frac{(\nabla_u G^T\boldsymbol{u})^2}{(\nabla_u G^T\nabla_u G)} \tag{a-8}$$

and

$$\beta = \frac{-\nabla_u G^T\boldsymbol{u}}{(\nabla_u G^T\nabla_u G)^{1/2}} \tag{a-9}$$

If $\boldsymbol{u}$ satisfies Equation (a-4) and Equation (a-7), the reliability index β can then be calculated from Equation (a-9).

To seek the solution vector $\boldsymbol{u}$, an iteration scheme is used [16]. This is based on a Taylor series expansion of G at a point $\boldsymbol{u}^*$ up to and including linear terms, G thereby being replaced by the tangent hyperplane at $\boldsymbol{u}^*$:

$$G(\boldsymbol{u}) = G(\boldsymbol{u}^*) + \nabla_u G^T(\boldsymbol{u}^*)(\boldsymbol{u} - \boldsymbol{u}^*) \tag{a-10}$$

By setting

$$G(\boldsymbol{u}^*) = \nabla_u G^T(\boldsymbol{u}^*)(\boldsymbol{u} - \boldsymbol{u}^*) = 0 \tag{a-11}$$

one obtains

$$\nabla_u G^T(\boldsymbol{u}^*)\boldsymbol{u} = \nabla_u G^T(\boldsymbol{u}^*)\boldsymbol{u}^* - G(\boldsymbol{u}^*) \tag{a-12}$$

Now applying Equation (a-7) to the tangent hyperplane,

$$\boldsymbol{u} = \frac{\nabla_u G^T(\boldsymbol{u}^*)\boldsymbol{u}}{\nabla_u G^T(\boldsymbol{u}^*)\nabla_u G(\boldsymbol{u}^*)} \nabla_u G(\boldsymbol{u}^*) \tag{a-13}$$

Combining Equations (a-3) and (a-13),

$$\boldsymbol{u} = \frac{-G(\boldsymbol{u}^*) + \nabla_u G^T(\boldsymbol{u}^*)\boldsymbol{u}^*}{\nabla_u G^T(\boldsymbol{u}^*)\nabla_u G(\boldsymbol{u}^*)} \nabla_u G(\boldsymbol{u}^*) \tag{a-14}$$

Similarly, for Equation (a-9), we have

$$\beta = \frac{G(\boldsymbol{u}^*) - \nabla_u G^T(\boldsymbol{u}^*)\boldsymbol{u}^*}{(\nabla_u G^T(\boldsymbol{u}^*)\nabla_u G(\boldsymbol{u}^*))^{1/2}} \tag{a-15}$$

Equations (a-14) and (a-15) provide the answer directly if the failure surface G were, in fact, the tangent hyperplane at $\boldsymbol{u}^*$. Since, in general, this is just an approximation, the calculated $\boldsymbol{u}$ is used as the new $\boldsymbol{u}^*$ and the procedure is repeated until convergence is achieved. The convergence of this quasi-Newton iteration scheme is not assured, however, as it is sensitive to the initial vector $\boldsymbol{u}^*$. The reliability index β is finally calculated from Equation (a-15) or (a-9).

In general, when the random variables $\boldsymbol{x}$ are correlated and non-Normal distributed, suitable transformations are required before the procedure is implemented, as described elsewhere [10].

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