Evaluation of the Constitutive Equation in COMPRO for Modulus Development

Project done by:
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Summary:
The Cure Hardening Instananeously Elastic (CHILE) formulation used in COMPRO is examined. A comparison is made between the results of a CHILE analysis with a full ViscoElastic (VE) analysis which shows an equivalence of the two approaches for most practical cases.

References:

*Johnston A, Vaziri R, Poursartip A. A Plane Strain Model for Process-Induced Deformation of Laminated Composite Structures. JCM 2001: 35(16); 1435-1469

* Zobeiry N, Rasekh A, Vaziri R, Poursartip A. Efficient Modelling Techniques for Predicting Processing Residual Stress and Deformation in Composite Parts. ICCM 14 2003

Why Does COMPRO use CHILE?

COMPRO uses a cure-hardening instantaneously linear elastic (CHILE) constitutive equation for modulus development, rather than a full visco-elastic (VE) formulation. The CHILE approach, which can be shown to be a pseudo-visco-elastic approach means:

- Decreased computational runtime which makes the analysis of complicated problems more tractable
- Encourages more frequent use of the models, in particular when exploring the response surface in preliminary design
- Encourages running bigger and more accurate models that better represent the real problem
- Efficient constitutive models can reduce the costs and effort required to characterize material parameters

Constitutive models in Process modeling

Models used in process modelling:

- Purely elastic models
- Hardening elastic models such as our CHILE (Cure Hardening Instantaneously Linear Elastic) model
- Viscoelastic models

Purely elastic models offer good insight, but do not capture enough of the complexity

Viscoelastic models should be the most accurate, however run-times for industrially relevant problems are prohibitively long

Hardening elastic models are fast and easy to use, and the resulting predictions appear to agree well with experiments

Viscoelastic Constitutive Models

Polymers show viscoelastic behavior at high temperatures, especially when they are partially cured

The existing VE models in the literature use the following hereditary integral formulation:

$$\sigma(t) = \int_0^t E\left(\xi(t) - \xi'(\tau)\right)\frac{d\xi}{d\tau}d\tau$$

$$\xi(t) = \int_0^t \frac{d\tau}{a_T(\alpha, T)}$$

is the reduced time variable based on time-temperature superposition

Hardening Elastic and Pseudo-Viscoelastic Models

A hardening elastic (CHILE) model is used in our code, COMPRO.

An elastic modulus that is a function of temperature and degree of cure is used: $E = E(T, \alpha)$

Previously, the frequency (or time) at which the modulus was evaluated was not based on any strict considerations.

However, if the elastic modulus in the CHILE model is taken as the VE storage modulus from a cyclic loading test at an appropriate frequency, or the VE relaxation modulus at an appropriate time, then the model is what we can call a 'Pseudo-Viscoelastic' (PVE) model and the resulting stress is:

$$\sigma(t) = \int_0^t E'(\alpha, t) \frac{d\varepsilon}{d\tau} d\tau$$

Comparison of Formulations

If the elastic modulus in PVE model is calibrated using the proper relaxation time, or the appropriate frequency, in dynamic viscoelastic tests, then the residual stress predictions using PVE and VE analyses should be very close.

- This can be shown by
  - numerical comparison (this presentation), and
  - by theoretical analysis and simplification of the VE constitutive equations (to be published)
- There are limits to the applicability of this PVE approximation, but it covers many cycles of interest.

Numerical Predictions

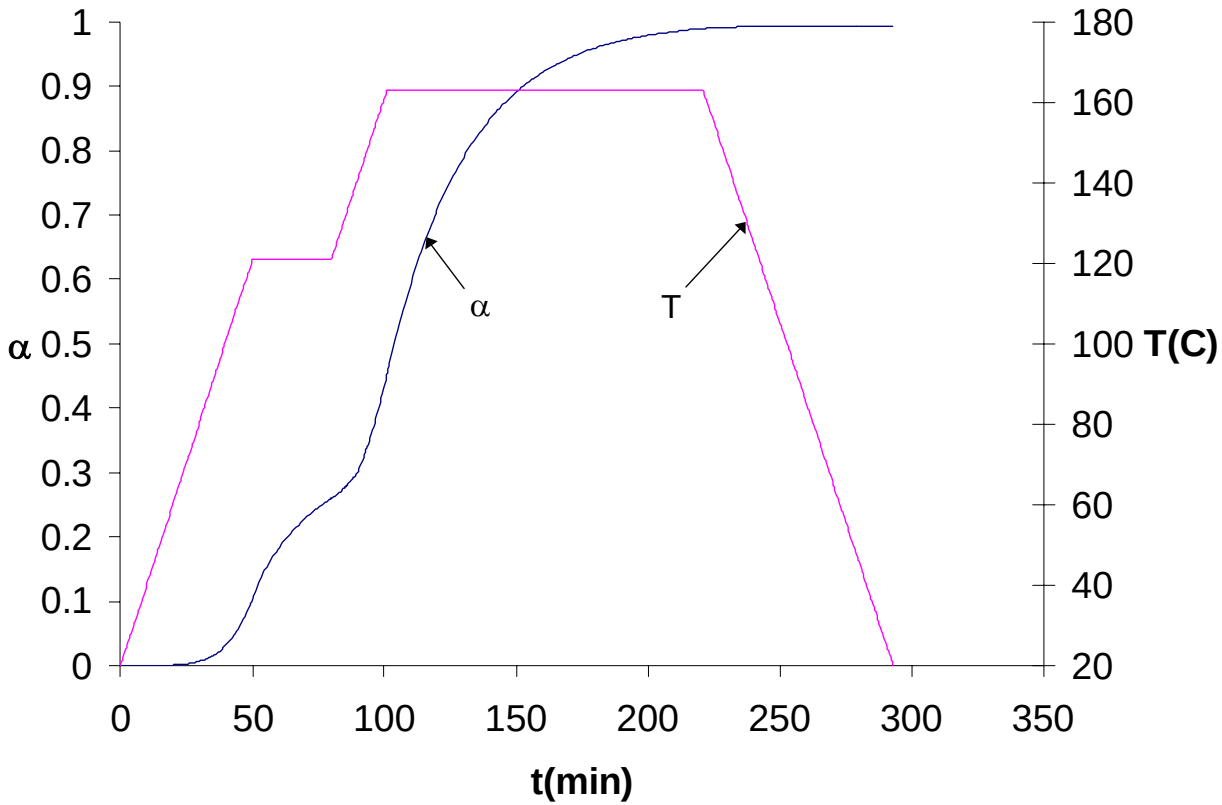
A 1D constrained 3501-6 epoxy resin structure undergoing a cure cycle is considered

Both thermal and cure shrinkage strains are included

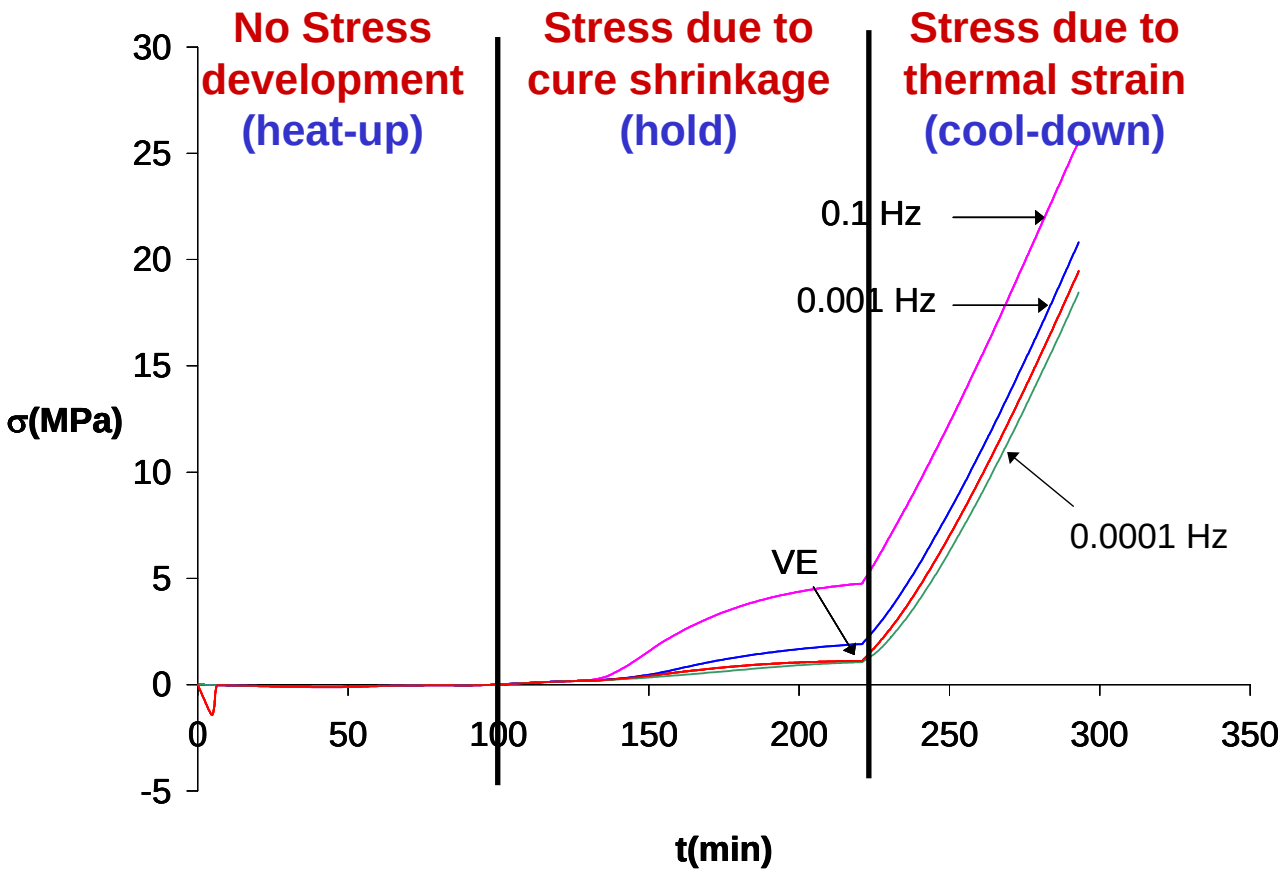
Kim & White (1996) data and formulations are used

The storage modulus is used as the elastic modulus

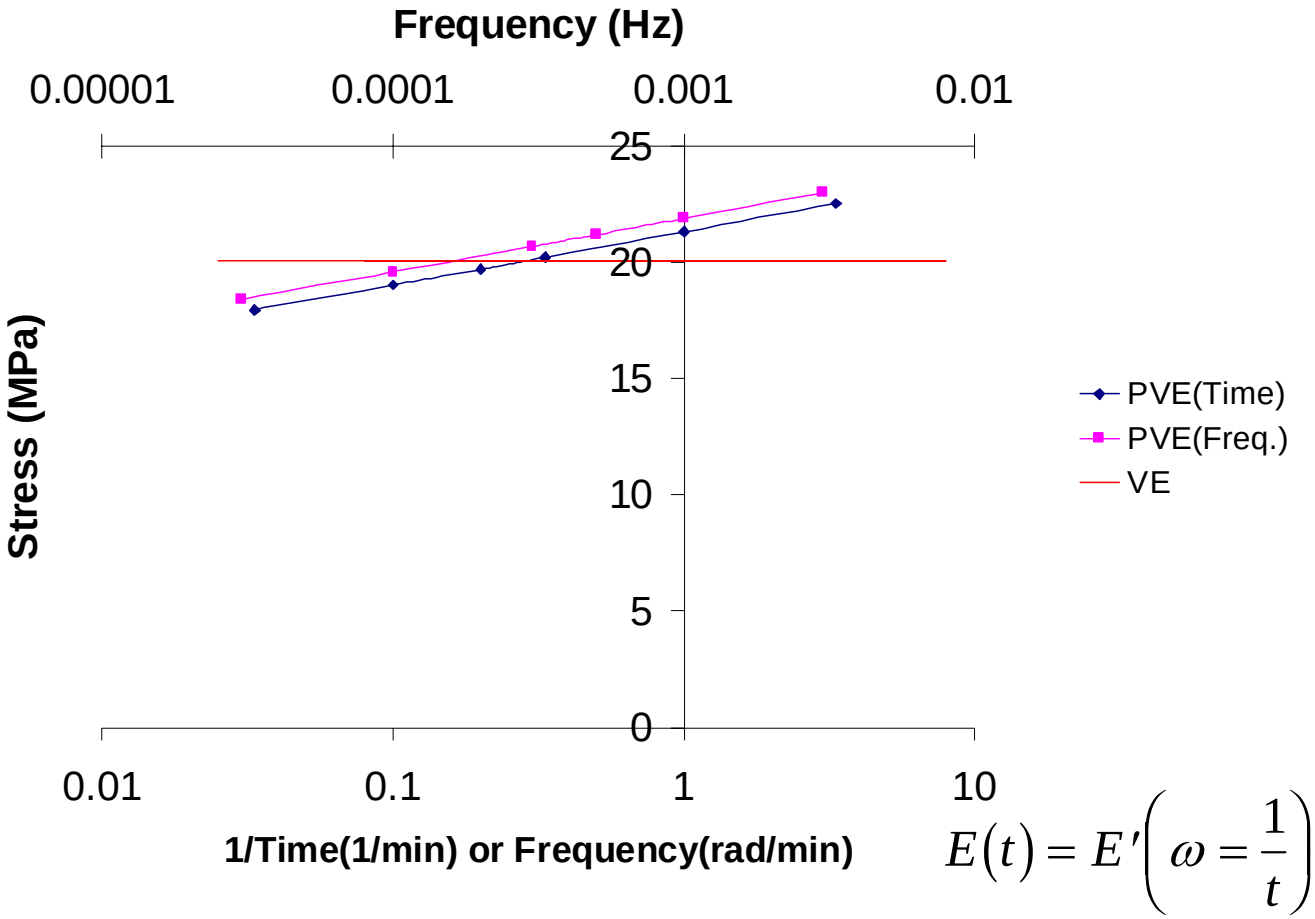
The Cure Cycle



Comparison of VE and PVE Predicted Stress Histories



Residual Stress



$$E(t) = E'\left(\omega = \frac{1}{t}\right)$$

Conclusions

- CHILE/PVE formulation is equivalent to a VE formulation for constitutive modeling of composite materials undergoing conventional cure cycles, given that the correct frequency or time is used
  - Because of the correlation of time and frequency based PVE formulations, both can be used in modeling.

Future work

Pseudo-Viscoelasticity

A process and procedure where the validity of using a PVE model (and calibrating the constants) can be established in advance of running the numerical cases.

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